
DSC 190 - Discussion 06

Problem 1.

Consider the following problem: given an $n \times n$ grid of unique numbers and a positive integer k , pick k numbers from the grid so that the sum of the numbers is maximized. Clearly, the greedy strategy of picking the k largest numbers is guaranteed to be optimal.

Argue that *any* optimal solution to this problem must include the largest number in the grid using a proof by contradiction.

Problem 2.

The *Fibonacci sequence* is a sequence of numbers where the next number is determined by adding the two previous numbers. The sequence is famous because the ratio of consecutive elements converges to the golden ratio.

The formal definition of the Fibonacci sequence is recursive:

$$x_n = \begin{cases} 1 & \text{if } n \in \{0, 1\} \\ x_{n-1} + x_{n-2} & \text{otherwise} \end{cases}$$

- a) As a warmup, write out the first six elements of the sequence, starting with $n = 0$
- b) Write a recursive function which takes in n and computes the n th Fibonacci number. How large can n be before your function starts to take more than a second or so to finish?
- c) Draw the recursion tree that results from running your function with $n = 5$. Are there overlapping subproblems?
- d) Your recursive code for computing the Fibonacci sequence is, in a sense, a *backtracking* algorithm. Modify it by adding a cache to create a top-down dynamic programming solution for computing the n th Fibonacci number.
- e) Create a bottom-up, iterative version of your function.