DSC 190 - Homework 05

Due: Monday, November 6

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 PM.

Problem 1.

In lecture, we considered the scheduling problem: given n pairs of event start and finish times, $[s_i, f_i)$, find a schedule containing the greatest number of non-overlapping events. We saw that the greedy strategy of picking events in order of their finish times is guaranteed to be optimal.

a) Given the following start and finish times, find an optimal schedule:

Event	Start	Finish
a	0	2
b	0	1
c	3	4
d	1	3
e	4	6
f	2	5
g	5	7

b) Just because a problem is solved by a greedy solution it does not mean that the greedy solution is the *only* solution.

True or False: in general, an optimal solution to the scheduling problem **must** contain the event with the earliest finish time.

If you answer "False", provide a counterexample in the form of an set of start/finish times and an optimal schedule that does not contain the event with the earliest finish time. If you answer "True", provide an argument that an optimal schedule must contain the event with the earliest finish time (e.g., with a proof by contradiction as demonstrated by the examples in this week's discussion).

Problem 2.

Consider the following problem: given an integer amount T, determine the minimum number of coins needed to make change for T cents using only quarters (25 cents), dimes (10 cents), nickels (5 cents), and pennies (1 cent).

For example, to make change for 55 cents, the minimum number of coins needed is three: two quarters and a nickel.

a) Describe a greedy algorithm for solving this problem. You may provide pseudocode or a description in words.

b) What is the minimum number of coins (quarters, dimes, nickels and pennies) necessary to make change for 131 cents? List how many of each coin is needed.

c) Suppose T is larger than 25 cents. True or False: the optimal solution must contain *exactly* $\lfloor T/25 \rfloor$ quarters.

If you answer "False", provide a counterexample in the form of a specific value of T and an optimal solution that does not contain exactly $\lfloor T/25 \rfloor$ quarters. If you answer "True", provide an argument

that an optimal solution must contain exactly $\lfloor T/25 \rfloor$ quarters (e.g., using a proof by contradiction as demonstrated by the examples in this week's discussion).

d) Now consider a more general version of the problem where coins have unspecified values $v_a > v_b > v_c > v_d$. It is **not** the case that the greedy algorithm will work here – it depends on what the coin values are.

Provide a set of coin values for which the greedy algorithm you described above will fail. Demonstrate that the greedy algorithm fails by writing down the solution it finds and showing that there is a better solution.

Problem 3.

Consider the following problem. You are a mail carrier in a one-dimensional universe. You are given a collection of points, $\{p_1, p_2, \ldots, p_n\}$ on a line (i.e., points in $\mathbb{R}^1$) which represent houses you must deliver mail to.

a) Describe a greedy algorithm which finds a mail route (a sequence of houses to visit) which minimizes the total distance walked. You may start and end your route at any location (you do not need to account for getting to your starting location – you can assume that you're teleported there).

b) Suppose the houses are located at the points: (3, 13, 1, 7, 25, 10). Give an optimal route for this set of points.

c) Assume that you start your route at the leftmost point (e.g., in the example above, you start at point 1) and that the house locations are unique. True or False: in any optimal route where the first stop is the smallest point, the second stop must be at the second smallest point.

You do not need to provide justification for this part.